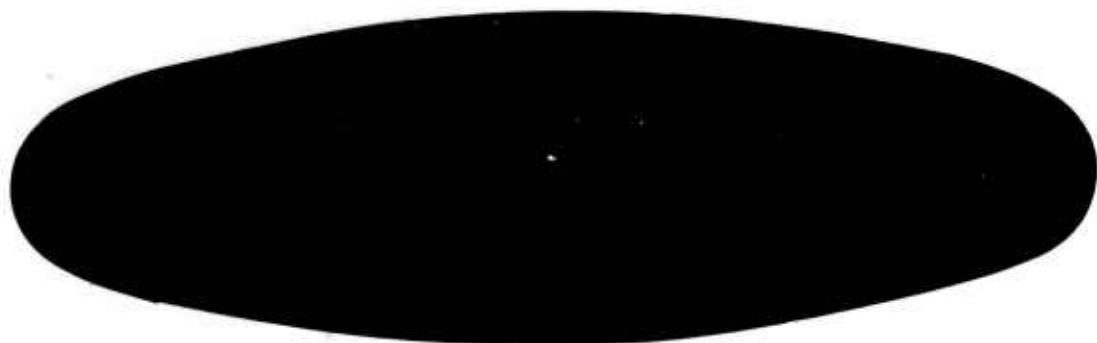


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DETECTING DEMAND CHANGES

By

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DETECTING DEMAND CHANGES

1. Introduction

One of the foremost problems facing the inventory manager is that of anticipating changes in demand patterns and adjusting the inventory level accordingly. If future demands were exactly known then of course there is no problem, for then the inventory level can be adjusted in anticipation of demand changes. The realistic case arises when demand is probabilistic over time, that is, a stochastic process. Even when one is willing to assume the mean value function of such a process as known, it is not possible to predict the exact demands and the best one can do is provide reasonable estimates of future demands. Then it is quite appropriate to adjust the inventory level in accordance with those estimates in such a way as to effect rapid response to changes that may occur as time progresses.

If the inventory level at any particular time is established as a result of analyzing the history of demands up to that time, then it is quite natural to view an inventory system as a servomechanism or "black box" in which the input is demand and the output is inventory. Such an approach to the study of inventory systems appears to have begun with Simon [4] and Vassian [5] in the early 1950's, was revised somewhat briefly by Howard [2] in 1963 and examined more recently by Reilly [3] and the writer. Except for a few isolated articles in the literature, no extensive application of the numerous tools developed in servo theory to inventory systems seems to exist. The purpose of this report is to direct attention to

these techniques as a means of analyzing and designing inventory control systems.

No attempt will be made in this preliminary report to obtain numerical solutions to a particular problem. Rather, attention will be devoted to structuring a model within which it is anticipated that several special problems can be solved. In particular, the basic problem of designing an inventory system that will be sensitive to demand changes will be borne in mind at all times. Of special concern is that of an item whose demand pattern changes from that of a slow mover to a moderate or fast moving item. Hopefully, there will be an opportunity in the future to process some real demand data from a Naval supply system to test the model and possibly suggest changes in either the model or the supply system as a result. In this early stage, relatively simple examples will be examined for further insight and for illustrative purposes.

2. Description of the Model

The inventory model we wish to adopt in this paper is a system with periodic review and fixed order points. Such a system is widely used in practical inventory situations where periodic checks of stock are far more economical than continuous review. It is one that has actually been used in the past by the military for spare parts. For the sake of convenience, we assume that order points are fixed and equally spaced. Without loss of generality, let the order points be designated by the positive integers on the time scale. We suppose a physically realizable system so that only the non-negative time axis is appropriate.

To be more specific, we assume that the t^{th} demand period is measured by real time between $t-1$ and t for $t = 1, 2, 3, \dots$. At the end of a demand period, and only then, a replenishment order θ_t is placed against future demands. We suppose a fixed lag time T in delivery where T is measured in demand periods so that an order placed at time t is delivered at time $t + T$ and is hence usable for fulfilling demands during the $(t+T+1)^{\text{st}}$ demand period. We further suppose that any oversupply at the end of a given period may be returned to vendor so that a replenishment order may be negative.

As to the assumptions about the inventory level, we suppose that the system begins with a safety level or initial inventory I_s at time zero. Since our primary concern will be whether or not the inventory level in future review periods is greater than or less than the safety level we shall henceforth, for the sake of simplicity, assume that the inventory level, I_t , at the end of the t^{th} demand period is the difference between the actual inventory and I_s at that time. Moreover, if demand should exceed on hand inventory we allow back orders to be placed against future deliveries. Thus, negative values of I_t are permissible in the analysis to follow.

As to the demand itself, the above conditions make it natural to assume that it can be described by a discrete parameter stochastic process. We will denote the total demand during the t^{th} period by X_t , $t = 1, 2, 3, \dots$ and we let $X_0 = 0$ for the sake of completeness. It will be helpful in the analysis if we suppose that $X_t = m(t) + \epsilon_t$ where $m(t)$ is the deterministic mean value function of the process $\{X_t\}$ and ϵ_t is a random variable with $E[\epsilon_t] = 0$ for all t . The process $\{\epsilon_t\}$ is sometimes

referred to as the noise component.

The mathematical model for the inventory system that we have described can now be nicely summarized by the recursive relation,

$$(2-1) \quad I_t = I_{t-1} + \theta_{t-1-T} - X_t, \quad t = 1, 2, 3, \dots$$

Such an inventory system has the feature that it is automatically controlled once we decide on the form of the reorder rule θ_t . We turn our attention to this matter in the next section.

3. Determination of the Reorder Rule

There are of course infinitely many ways to specify a reorder rule consistent with our model. To choose one as best requires some criterion for choice. An obvious criterion in the present case is to require an order rule which will minimize the inventory variations about the safety level. It is not surprising that a solution among all possible choices of θ cannot exist and we must restrict our order rules to lie in some special class of functions if we hope to find a solution. A natural restriction is to consider only those rules which depend upon past demands and inventory levels. To this end, we restrict our attention to only those order quantities that can be expressed as a linear combination of past demands and inventories. However, as time progresses, we would certainly want to weight the early part of our history with smaller and smaller values. This suggests that we express a general reorder rule as follows,

$$(3-1) \quad \theta_t = \sum_{j=0}^t A_j X_{t-j} + \sum_{j=0}^t B_j I_{t-j}, \quad t = 1, 2, 3, \dots$$

The problem is thus reduced to that of determining the sequences $\{A_0, A_1, A_2, \dots\}$ and $\{B_0, B_1, B_2, \dots\}$. Observe that the same weights are always given to the most recent observations while the weights given the early observations vary with time. Of course X_0 and I_0 are both zero but it does no harm to write them in the expression for θ_t and makes for a certain degree of completeness in the sequel.

Even with the additional restrictions it is not possible in general to find a unique order rule. Vassian [5] has given a partial solution to the problem in the sense of providing a rule which depends upon one's ability to forecast, at each time t , the total demand over the following $T+1$ periods, i.e., demands during periods $t+1$ through $t+T+1$. Indeed, this is about as close to a solution as one would expect to arrive, Vassian's claim for a solution notwithstanding, because of the random nature of demand. At any time t , past demands are of course known but future demands, hence inventories, are random. Consequently, the concept of minimizing inventory should be replaced with that of minimizing expected inventory and we require that at least $\lim_{t \rightarrow \infty} E[I_t] = 0$ for any specified order rule. It is then possible to find a family of order rules which will satisfy equations (2-1) and (3-1) subject to this condition. The details of the solution are important enough to be repeated here, not only to see how a solution is effected, but also to gain insight into the nature of our model as a servomechanism.

Because of the recursive nature of equation (2-1) and the fact that both expressions in equation (3-1) are recognizable as the convolution of

sequences, it is clear that our conditions may be converted by means of z-transforms into equivalent and more algebraically tractable form. To this end, if V represents any sequence $\{v_0, v_1, v_2, \dots\}$ we will denote the corresponding z-transform, $\sum_{k=0}^{\infty} v_k z^k$ by $V(z)$. Recalling that the z-transform of the sequence $\{v_{k+r}\}_{k=0}^{\infty}$ is, (for any integer r) $z^{-r}V(z)$ and that the z-transform of the convolution of two sequences is given by the product of their transforms, we can now rewrite equations (2-1) and (3-1) respectively as,

$$(3-2) \quad I(z) = zI(z) + z^{T+1}\theta(z) - X(z), \quad \text{and}$$

$$(3-3) \quad \theta(z) = A(z) X(z) + B(z) I(z)$$

By substituting the expression for $\theta(z)$ from (3-3) into (3-2) the inventory transform can be written,

$$(3-4) \quad I(z) = \frac{(z^{T+1}A(z)-1)}{1-z-z^{T+1}B(z)} \cdot X(z)$$

Or, letting $S(z) = \frac{z^{T+1}A(z)-1}{1-z-z^{T+1}B(z)}$, we can write, more simply,

$$(3-5) \quad I(z) = S(z) X(z)$$

thereby expressing the inventory at any time t as a convolution of a sequence $\{S_0, S_1, S_2, \dots\}$ with past demands. Because of (3-5), it is quite appropriate to call $S(z)$ a transfer function and to view inventory

as the output of a "black box" the input of which is demand which is operated on by the transfer function. Since $S(z)$ depends only upon $A(z)$ and $B(z)$ we can define the latter to satisfy our criterion of minimizing expected inventory by appealing to well known results from the theory of servomechanisms.

Since we can write $I_t = \sum_{j=0}^t S_j X_{t-j}$ where S_j is non-random, we see that $E(I_t) = \sum_{j=0}^t S_j E(X_{t-j}) = \sum_{j=0}^t S_j m(t-j)$ for $t = 1, 2, 3, \dots$. Letting $E_t = E(I_t)$ denote the expected inventory at time t , we see that $E(z) = X(z) m(z)$ in terms of the same transfer function $S(z)$. Now, since $m(z)$ is deterministic, the requirement $\lim_{t \rightarrow \infty} E_t = 0$ is equivalent to requiring the poles of $S(z)$ to be outside the unit circle. Since $A(z)$ only occurs in the numerator of $S(z)$, the latter requirement depends only upon the choice of $B(z)$. As Vassian suggests, if we let $B(z) = -(1-z)/(1-z^{T+1})$, then $S(z)$ will have no finite poles. Indeed, with this choice of $B(z)$, it is easy to see that $S(z)$ can then be written as $(z^{T+1}A(z)-1) \frac{(1-z^{T+1})}{1-z} = (z^{T+1}A(z)-1) \sum_{k=0}^T z^k$ which becomes infinite only as z does. We have thus been able to determine one of our unknown sequences, though the choice is by no means unique.

As to the determination of $A(z)$ we next observe that we may now write $I(z) = z^{T+1} A(z) \frac{(1-z^{T+1})}{1-z} X(z) - \frac{(1-z^{T+1})}{1-z} X(z)$. But $\frac{1-z^{T+1}}{1-z} = \sum_{k=0}^T z^k$ is the z-transform of the sequence $\{U_k\}_{k=0}^{\infty}$, where $U_k = 1$ if $k \leq T$; otherwise, $U_k = 0$. Letting $A^*(z) = z^{T+1} A(z) \frac{(1-z^{T+1})}{1-z} X(z)$, we have, from equations (3-4) and (3-3), $I(z) = A^*(z) - \frac{(1-z^{T+1})}{1-z} X(z)$

and $\theta(z) = \frac{1-z}{1-z^{T+1}} \frac{A^*(z)}{z^{T+1}} - \frac{1-z}{1-z^{T+1}} I(z)$. Finally, we may write

$$(3-5) \quad \frac{1-z^{T+1}}{1-z} \theta(z) = z^{-(T+1)} A^*(z) - I(z), \text{ and}$$

$$(3-6) \quad A^*(z) = I(z) + \frac{1-z^{T+1}}{1-z} X(z)$$

Or, in equivalent form, if $t \geq T$,

$$(3-7) \quad \sum_{j=0}^T \theta_{t-j} = A^*_{t+1+T} - I_t \quad \text{and}$$

$$(3-8) \quad A^*_{t+1+T} = I_{t+1+T} + \sum_{j=0}^T X_{t+1+T-j} = I_{t+1+T} + \sum_{j=t+1}^{t+1+T} X_j$$

Now, from (3-8) $E(A^*_{t+1+T}) = E_{t+1+T} + \sum_{j=t+1}^{t+1+T} m(j)$. Since we want

$\lim_{t \rightarrow \infty} E_t = 0$, this requires that we choose A^*_{t+1+T} , in such a way that $E(A^*_{t+1+T}) - \sum_{j=t+1}^{t+1+T} m(j) \rightarrow 0$ as $t \rightarrow \infty$. Let us call A^*_{t+1+T} a forecast

of the demand from period $t+1$ through period $t+1+T$, i.e., over the following $T+1$ periods.

We have thus reduced the problem to one of determining a suitable means of forecasting future demands. "Suitable" here means that the forecast must be at least asymptotically accurate in the sense of the above limiting relation. We may now express our order rule in the form

$$(3-9) \quad \theta_t = A_{t+1+T}^* - \sum_{j=1}^T \theta_{t-j} - I_t$$

where A_{t+1+T}^* is any function of the process X_t such that

$E(A_{t+1+T}^*) - \sum_{j=t+1}^{t+1+T} m(j) \rightarrow 0$ as $t \rightarrow \infty$. Of course, once A_t^* is specified

we can, at least in principle, determine the sequence A_t from the definition of $A^*(z)$. This is not required for the order rule, however, which now states that at time t , one orders an amount equal to the forecast of demand over the next $T+1$ units of time less those orders placed at times $t-1, t-2, \dots, t-T$ (which will be receipts at times $t+1, t+2, \dots, t+T$) and, of course, less the inventory at time t .

All that remains for a given problem is to determine a forecaster, or equivalently, the function $A^*(z)$. No further explicit expressions can be derived until more assumptions are made and, quite obviously, the determination of A^* will not be unique.

4. The Model as a Servomechanism

The general inventory system which we have described, first by equations (2-1) and (3-1) and then, in a transformed version, by means of equations (3-2) and (3-3), can be considered a discrete-variable servomechanism. Indeed, as we have brought out in the preceding section, particularly with equation (3-5), the inventory may be thought of as the output of a system whose input is demand. In fact, in the present case, the system resembles an automatic control device with a feedback system which allows for taking past history into account at each decision-making

stage. These facts are borne out more clearly by standard servomechanism diagrams. Moreover, such diagrams often make it possible to replace one system by an equivalent one for purposes of further analysis.

A few remarks concerning the meaning attached to various graphical forms in systems analysis may be helpful. First of all, whenever two transforms are connected in series, we will always understand the output to be the product of the two quantities. Thus, the basic product relationship of our system expressed by equation (3-5), i.e., $I(z) = S(z)X(z)$ can be portrayed graphically as in Fig. 4.1.

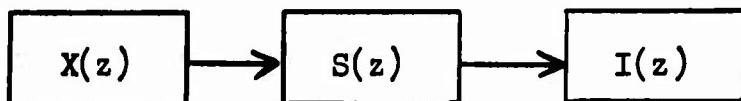


Figure 4.1: Graph of $I(z) = S(z)X(z)$

On the other hand, whenever two quantities are connected in parallel, the result, usually depicted as a circle or node at the junction of two directed paths, is taken to be the sum of the two quantities as shown in Figure 4.2.

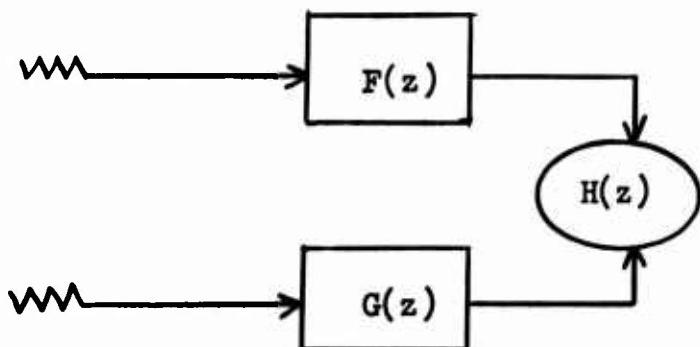


Figure 4.2: Graph of $H(z) = F(z) + G(z)$

With these conventions in mind, the inventory system described by equations (3-2) and (3-3) can be represented as in Figure 4.3. In this form, the

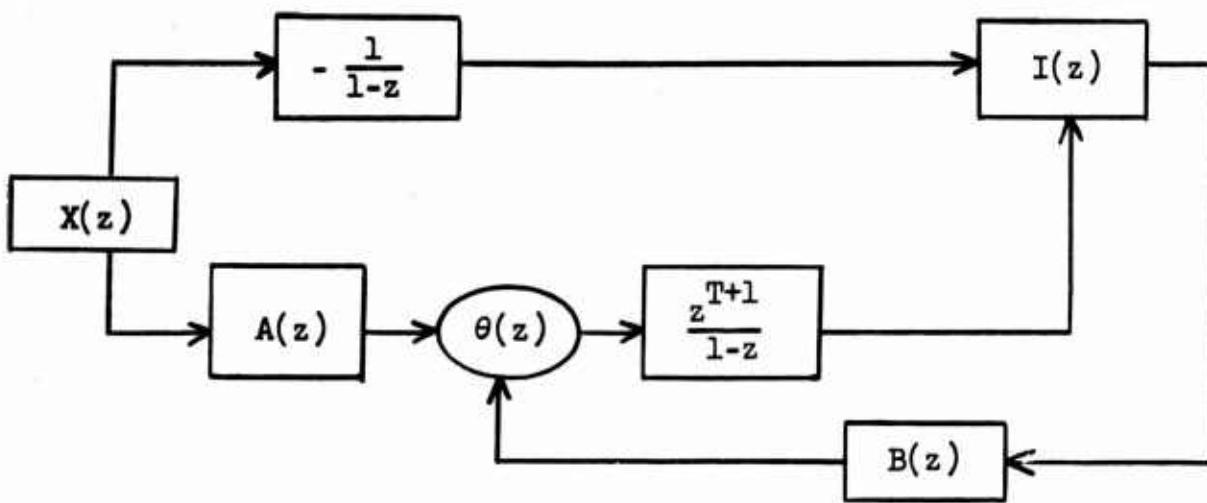


Figure 4.3: The Inventory System

feedback property of the order rule is dramatically displayed.

In order to obtain a graphical view of the role played by the forecaster $A^*(z)$ of our last section, it is necessary to replace the original system represented by Figure 4.3 by an equivalent one. This is accomplished by means of repeated applications of the two basic rules for displaying sums and products, respectively, as parallel and series connections. The results are displayed in Figure 4.4 which displays an equivalent system free of loops.

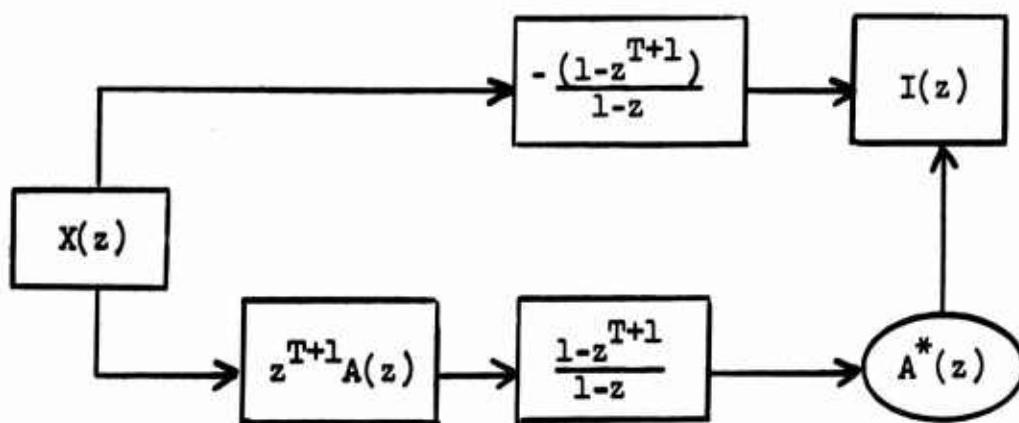


Figure 4.4: Inventory System with Forecast

Many other equivalent systems may be formed, depending on the various aspects of the system that need to be analyzed. Also, a change from one form of pictorial representation to another often suggests definitions of quantities that may add to a deeper understanding of the system.

Once a flow graph of a system is determined as in Figure 4.3 it is possible to obtain the system transfer function $S(z)$ by means of a few conventional definitions in a rather easy manner. Indeed, the method often is a distinct advantage over the analytical solution which we determined in the last section.

First of all, we shall refer to any function denoted by a square between input and output as a branch transfer function. We have already referred to circled functions as nodes and a path is always indicated by arrows. A loop is a closed path in the system and one or more loops always occur in feedback systems. For any simple path between input and output, the path transmission, $P(z)$ is the product of all branch transfer

functions in the path. For a loop, the product of all branch transfer functions in the loop is called the loop product. Finally, the system determinant $\Delta(z)$ is defined to be unity minus the sum of all loop products in the system. Each path with transmission $P_j(z)$ in turn has a path determinant $\Delta_j(z)$ defined as unity plus the loop product of all loops in the system which have no node in that path. With these definitions, it turns out that the transfer function $S(z)$ can be easily expressed by means of the equation,

$$(4-1) \quad S(z) = \frac{\sum_j P_j(z) \Delta_j(z)}{\Delta(z)}$$

In the present case, using Figure 4.3, there is a single loop with product $\frac{z^{T+1}B(z)}{1-z} = -\frac{z^{T+1}}{1-z^{T+1}}$ and hence the system determinant is given by $\Delta(z) = 1 + \frac{z^{T+1}}{1-z^{T+1}} = \frac{1}{1-z^{T+1}}$. There are two simple paths with respective path transmissions and path determinants given by, $P_1(z) = \frac{-1}{1-z}$, $\Delta_1(z) \equiv 1$ and $P_2(z) = \frac{z^{T+1}A(z)}{1-z}$, $\Delta_2(z) \equiv 1$. Hence we have $S(z) = \frac{[z^{T+1}A(z)-1](1-z^{T+1})}{1-z}$ as we previously found.

5. Special Cases

One of the primary purposes of finding the system transfer function $S(z)$ as discussed in Sections 3 and 4 is to be able to examine the system response to various demand inputs. Now, with the inventory system expressed in the form $I(z) = S(z) X(z)$, once it is assumed that the

demand is random, it follows that the inventory level is also random. When this is the case, it is of course not possible to exercise deterministic control over the inventory level. For one thing, there can be no explicit expression for the demand input function $X(z)$ in this case. However, we saw in Section 3 that the expected inventory level responds to average demand in a precisely analogous fashion, i.e., $E(z) = S(z)m(z)$ is a system with exactly the same transfer function and a deterministic input given by $m(z)$, the z-transform of the mean value function of the process. With this in mind, we now proceed to examine the response of expected inventory to average demand in some special instances.

Certainly one of the simplest cases we can examine is that of a constant mean value, i.e., we suppose that $m(t) \equiv a$ where $a \geq 0$. For such a case then $m(z) = \frac{a}{1-z}$. What is the response of the system to such an input? In order to answer the question it is necessary to know the transfer function explicitly and this in turn depends upon the particular method of forecasting adopted.

Because of the basic feedback property of the system we have defined, every bit of past history is available whenever a decision on the order quantity must be made. It would therefore seem natural that we would want any method of forecasting we adopt to be efficient in the sense of utilizing all of the information available at a given order time. One such method is that of exponential smoothing developed somewhat extensively by Brown [1] and others. With this method of estimation, some credit is always given to each past demand with the least credit going to the oldest demand. Moreover, as time progresses, the credit allowed for early demands becomes negligible.

If we apply exponential smoothing to the present case, then, at any time t , we must forecast the demand over the next $T+1$ periods. Since our best estimate of demand at any future period is given by mean demand which in turn is the constant a , the smoothed estimate of a multiplied by $T+1$ will do. For the demand history x_1, x_2, \dots, x_t , the exponentially smoothed estimate of a is given by $\hat{a} = \alpha \sum_{k=0}^t \beta^k x_{t-k}$ where α is the chosen smoothing constant (typically 0.1 to 0.3) and $\beta = 1 - \alpha$. We define our forecast as $A_{t+T+1}^* = (T+1)\hat{a}$ and observe that $E(A_{t+T+1}^*) = (T+1)E(\hat{a}) = (T+1)\alpha \sum_{k=0}^t \beta^k E(x_{t-k}) = a(T+1)(1-\beta^{t+1})$ and $\sum_{j=t+1}^{t+T} m(j) = (T+1)a$ in the present case so that $E(A_{t+T+1}^*) - \sum_{j=t+1}^{t+T} m(j) = -(T+1)\beta^{t+1} \rightarrow 0$ as $t \rightarrow \infty$. Consequently, A_{t+T+1}^* satisfies our basic requirement of asymptotic accuracy for a forecast as was established in Section 3.

It is instructive to observe that in the present case, defining

$$A_{t+T+1}^* = (T+1)\hat{a} \text{ is equivalent to letting } A(z) = \frac{\alpha(T+1)(1-z)}{z^{T+1}(1-\beta z)(1-z^{T+1})}.$$

Then the explicit transfer function is determined as

$S(z) = \frac{\alpha(T+1)}{1-\beta z} - \frac{(1-z^{T+1})}{1-z}$. From servo theory, the steady state behavior of the system $E(z) = S(z)m(z)$ is defined as $\lim_{t \rightarrow \infty} E_t$ and may be computed in the transform domain as $\lim_{z \rightarrow 1} (1-z)E(z)$. In the present case, $(1-z)E(z) = S(z)$ and, since $\lim_{z \rightarrow 1} S(z) = (T+1) - (T+1) = 0$, it follows that $\lim_{z \rightarrow 1} (1-z)E(z) = 0$, verifying the fact that $\lim_{t \rightarrow \infty} E_t = 0$. In such a case, we say there is no steady state error.

As a second example, let us consider the problem mentioned in Section 1 of accounting for changes in demand when an item changes from a slow mover to a moderately fast mover to eventually a fast mover. A first approximation to describing this situation in the present model is to assume that $m(t) = bt$ for some small positive constant b . For such an item, mean demand would be negligible at first and would begin to become serious at some future time depending on b .

Of course we do not know the value of b , but then our inventory system is set up in such a way as to respond to changes in demand as we go along in time. Indeed, we appeal to exponential smoothing once more as a means of estimating b at each point in time utilizing all of the past data. Other techniques such as least squares might also be used to estimate b . One of the reasons we prefer exponential smoothing to least squares is the fact that far less information has to be stored in order to compute a running estimate as shown in Brown [1].

For the present case, Brown [1] and Reilly [3] have shown that, for a demand history of X_1, X_2, \dots, X_t , an appropriate estimator for b , based on exponential smoothing, is given by $\hat{b} = \frac{\alpha}{\beta} [\hat{X}_t - \hat{X}_{t-1}]$ where

$\hat{X}_t = \alpha \sum_{k=0}^t \beta^k X_{t-k}$ and $\hat{X}_t = \alpha \sum_{k=0}^t \beta^k \hat{X}_{t-k}$. Here again, α is the smoothing constant and \hat{X}_t is often referred to as the double exponential smoothing operator. Reilly [3] has shown that $\lim_{t \rightarrow \infty} E(\hat{b}) = b$ so that \hat{b} is asymptotically unbiased.

Because of the above results, we define $A_{t+1+T}^* = \sum_{k=t+1}^{t+1+T} \hat{b}_k$ as our forecast at time t of demand over the next $T+1$ periods. Now, for t sufficiently large, $E(A_{t+1+T}^*) = b \sum_{k=t+1}^{t+1+T} k$ and, of course

$$\sum_{k=t+1}^{t+1+T} m(k) = b \sum_{k=t+1}^{t+1+T} k \text{ so that } E(A_{t+1+T}^*) - \sum_{k=t+1}^{t+1+T} m(k) \rightarrow 0 \text{ as } t \rightarrow \infty$$

as required. If we use this procedure, we can be sure once again that our system $E(z) = S(z) m(z)$ has no steady state error. Indeed, the requirement that we adopt an asymptotically accurate forecaster was imposed for precisely this reason.

6. Summary and Recommendations

We believe that the theory of servomechanisms provides some powerful tools for analyzing dynamic inventory systems. The present report has been devoted to making a case for that point by treating one particular model as a discrete-variable servomechanism with automatic feedback. Servo techniques were helpful in finding explicit solutions by converting recursion relations to z-transforms where the mathematics was more tractable. Even more important, the graphical representations are extremely helpful in obtaining an overview of the system as a whole, one which is otherwise often difficult to realize.

There are several directions in which recommendations for further research in this area can be made. As previously mentioned, it would be of great practical interest to process a real inventory system in which the present model applies. Using real data, numerical answers could then be analyzed toward supporting or suggesting changes in the model. Also, the examples treated in this report are relatively simple ones - perhaps oversimplified for practical use - in which case further effort should be devoted toward applying the same sort of analysis to more complicated inputs.

As regards the model itself, changes can always be suggested that would typically complicate the analysis but approach reality more closely. For example, we have treated the lag in delivery as deterministic whereas it is typically random in reality. The model could be changed to a continuous review system in which case z-transforms would be replaced by Laplace transforms. Presumably, an analysis similar to the one we have outlined would apply to that case.

Another point to be made is that the solutions we have arrived at are explicit only up to specifying a forecast of future demands at any given time. Now there are many ways in which such a forecast might be specified even in the isolated examples we have treated. Exponential smoothing was the technique adopted in this report for the particular reasons mentioned. Nevertheless, other methods of forecasting might well compare more favorably on further reflection of the desired output in the model.

We have imposed the restriction that forecasters be asymptotically accurate. This may be too severe and, in any case, one without that property may very well do better in the early stages than one meeting the requirements. If such forecasters were to be admitted, then explicit expressions would have to be derived for the inventory level at any given time and the various techniques for inverting z-transforms would very likely play an important role in such an analysis.

As a final remark it should be observed that there is a natural extension of the servo techniques discussed in this report to vector-valued functions. This suggests that a more complex multi-echelon model might possibly be treated from an analogous point of view as given here.

Since this is a preliminary report, all of the above remarks may be taken as suggestions for further research in this area.

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A periodic review inventory system is viewed as a servomechanism in which the input is demand and the output is the inventory level. Under the assumptions that lead time is fixed and demand is random, an order rule is determined which minimizes the long run average inventory. The order rule allows for any method of forecasting future demands and completely determines the operating doctrine for each such method. Graphical portrayal is used to illustrate the essential features of the model in a servomechanism setting. Special cases, in which mean demand is specified and exponential smoothing is adopted as a forecasting technique are examined in detail. In particular, one of these examples treats of the case where an item changes from a slow mover to a moderately fast to fast mover.

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14.	KEY WORDS	LINK A		LINK B		LINK C	
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